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Merging Deep Learning & Functional Data Analysis for 3D Data Reconstruction in the Medical Context

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Motivation

Medical conditions can be diagnosed and studied using image data. For instance, Figure 1 displays an example of a 3D Magnetic Resonance Imaging (MRI) scan of a human brain. Standard medical software can reconstruct a model of the brain surface stored as a 3D mesh, as depicted on the left-hand side of Figure 2. Additionally, measurements of various quantities of interest are recorded on the mesh. For instance, Blood Oxygenation Dependent Level (BOLD) is used to infer the activity of brain cells. BOLD has an enormous range of uses, from a diagnostic tool in the hospital or clinic, to research into how the brain works, to investigating changes that occur in disease.

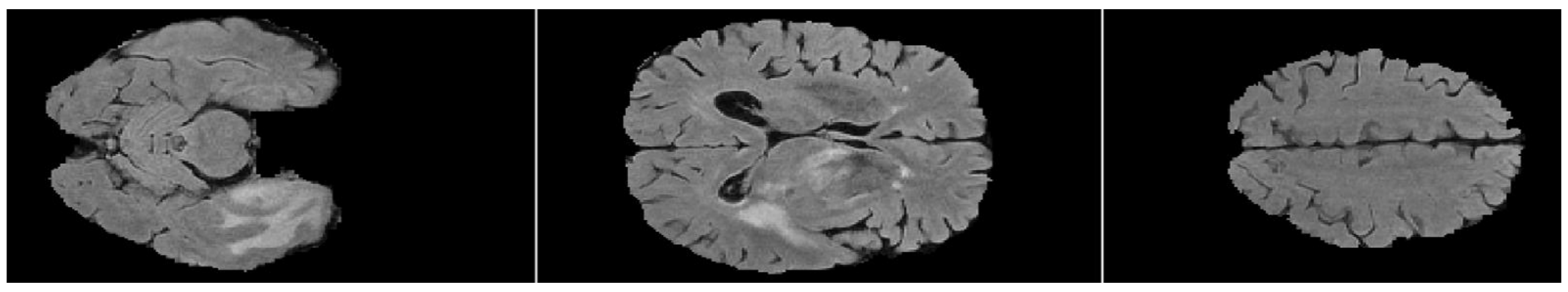

Figure 1: An example of an MRI scan of a human brain. The image consists of a three-dimensional array of voxels. Vertical cross-sections are shown from left to right.

Objectives

Medical image acquisition and preprocessing often introduces noise that contaminates the data. Recovering the underlying signal is crucial for an accurate analysis. The objective is to estimate the BOLD signal mapped onto the subject-specific cerebral cortex geometry, as depicted on the right-hand side of Figure 2. However, achieving reliable results requires novel methods that account for the intricate geometry and complex surface of the human brain.

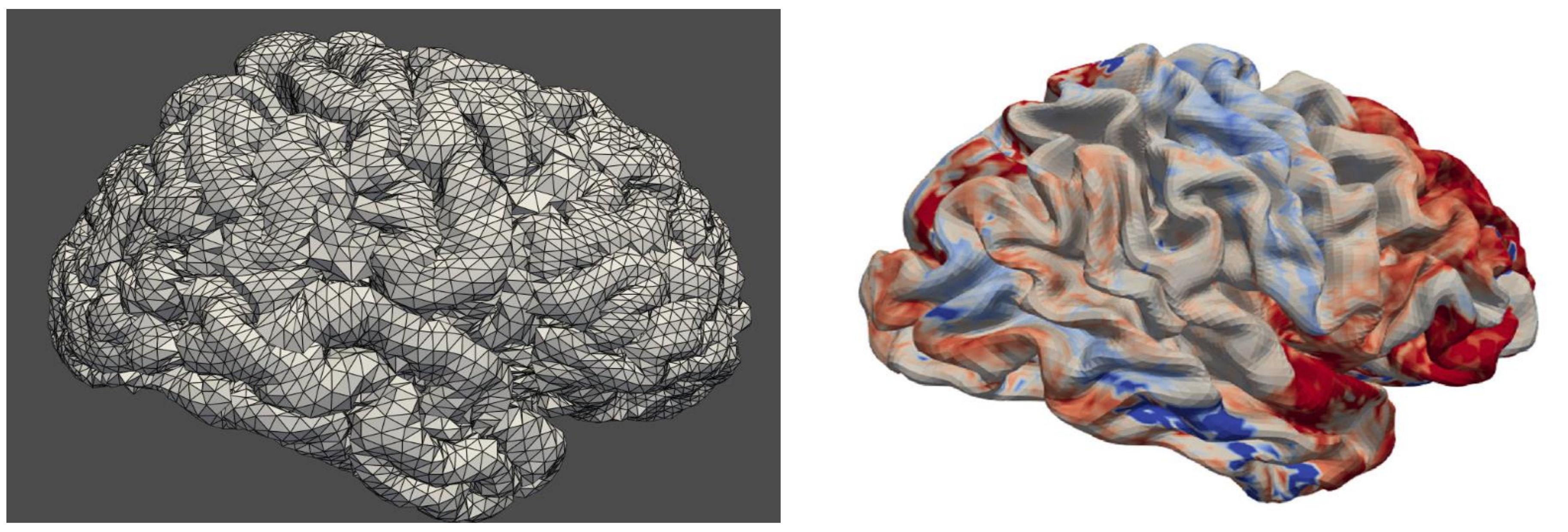

Figure 2: Left: A typical mesh of a human brain that can be extracted from an MRI scan. Right: BOLD signal mapped on top of the brain mesh.

Object Oriented Functional Data Analysis

Let x_i, y_i, z_i for $i = 1, \cdots, N$, denote N locations on the mesh and let s_i for $i = 1, \cdots, N$ be the noisy data values at these locations. Functional Data Analysis (FDA) assumes there is a continuous smooth function $f(x_i, y_i, z_i)$ defined on the cerebral cortex geometry giving rise to the data. That is $s_i = f(x_i, y_i, z_i) + \varepsilon_i$, where ε_i represents the measurement error. Combining the following methods allows our model to recover an accurate estimate of the underlying signal f from the observed data:

1. Regression with a Partial Differential Equation Regularization [1]

$$\mathcal{L}[f] = \sum_{i=1}^{N} (s_i - f(x_i, y_i, z_i))^2 + \lambda \int_{\Omega} (\nabla^2 f)^2 \, d\Omega,$$

where

- λ controls the degree of smoothing
- $\nabla^2 f$ measures the curvature of f with the Laplace-Beltrami operator.

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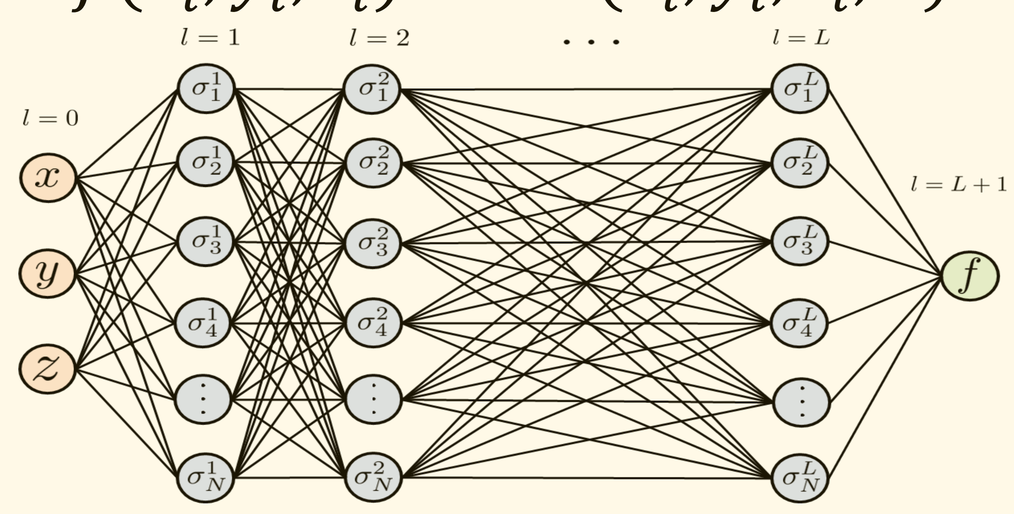
2. Physics-Informed Neural Network (PINN) [2]

$$\hat{f}(x_i, y_i, z_i) = \mathrm{NN}(x_i, y_i, z_i, \theta)$$



Results

A simulation of a real brain mesh was used to assess the method's performance. Figure 3's left-hand side displays an artificial signal $\boldsymbol{f(x,y,z)} = \prod_{i=1}^{3} \boldsymbol{cos}(\boldsymbol{\pi}(\frac{r_i - r_i^{min}}{r_i^{max} - r_i^{min}}))$, where $r_i = \{x, y, z\}$ and $r_i^{\{min,max\}} = \{min, max\}(r_i)$, mapped on top of the brain. In the center of Figure 3, random noise $\epsilon \sim N(0, \sigma^2)$ with $\sigma = 0.5$ is added to the signal. The right-hand side of Figure 3 presents the reconstructed signal using our model (PINN). For the proposed method (PINN) and two existing methods fdaPDE [1] and the Heat Kernel (HK) [3], we measured the Root-Mean-Square Error (RMSE) between the true and reconstructed signals. Figure 4 displays the distribution of the RMSE metric for 50 simulations. PINN performs on average **at least 60%** better than the existing approaches in the literature.

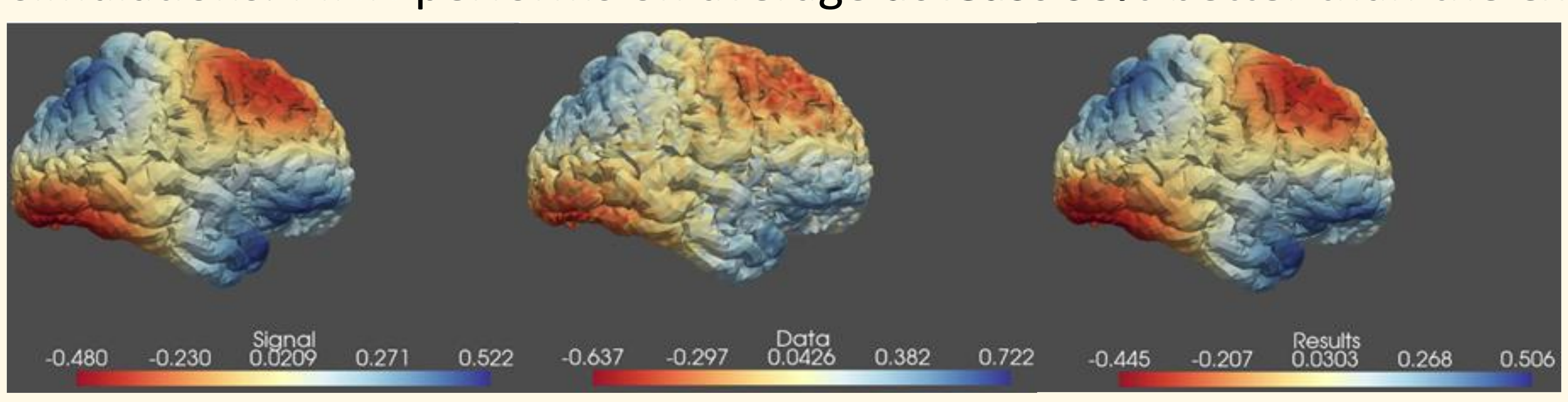


Figure 3: Simulations on a real human brain mesh. The brain surface is shown on the left with a simulated signal. Center: Random noise is added to the signal. The goal is to recover the left image from the center one. Right: Results of the reconstruction by out proposed method.

Figure 4: Performance of PINNs against standard methods in the FDA literature. The boxplot shows the distribution of the Root-Mean-Squared Error (RMSE) between the true signal and the reconstructed signal for 50 simulations.

References

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[3] Chung, Moo K., et al. "Cortical thickness analysis in autism with heat kernel smoothing." NeuroImage 25.4 (2005): 1256-1265.

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